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DRAFT TRANSLATION

PRESERVATION OF THE MAXWELLIAN DISTRIBUTION FORM IN A RELAXING GAS

(O sokhranenii formy maksvellovskogo raspredeleniya v relaksiruyushchem gaze)

Vestnik Moskovskogo Universiteta
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by D. I. Osipov

ABSTRACT.

The process of Maxwellian equilibrium settling in a light gas surrounded by an atmosphere of heavy gas is being investigated. It is assumed that the interaction forces are inversely proportional to the fifth power of the distance. It is shown that the initial light gas distribution being Maxwellian (with temperature differing from that of a heavy gas), the Maxwellian form of distribution in that gas shall be preserved in the equilibrium settling process. Only the temperature will change. The question of extending this result to other laws of interaction is also examined.

COVER-TO-COVER TRANSLATION

In the relaxation process of a gas toward equilibrium, the distribution function changes its form. If the initial distribution function has an equilibrated form, the relaxation process is not expected to alter it under certain specific conditions. Thus, the relaxation process of an oscillating flow in a system of harmonic oscillators placed in a thermostat, will be taking place in such a fashion, that the oscillators' distribution function, while remaining of the strictly Boltzmann type, will only modify its oscillating temperature [1].

inasmuch as the initial distribution of oscillators according to levels of oscillation is a Boltzmann type (with temperature differing from that of the thermostat). Another similar example — that of the preservation of Maxwellian distribution in the relaxation process — is examined in the present work.

Let us examine a system consisting of a large number of heavy atoms with mass M and a small addition of light particles and electrons with mass m. For the sake of simplicity, we shall only mention electrons in the following. At the initial time instant the temperature of heavy particles (thermostat) is T, and that of electrons — T_0. We shall consider that in the process of relaxation the temperature of the thermostat does not vary. We shall select the temperatures T and T_0 in such a way that all non-elastic processes, linked with the formation of negative and positive ions, may be neglected. In that case the settling of equilibrium in the electron gas will only take place as a result of energy exchange at elastic collisions of electrons with the atoms of the thermostat.

The behavior of the electron distribution function $f(v, t)$ in the relaxation process is described by the Boltzmann integral-differential equation. If we account only the elastic collisions of electrons with atoms, and if we consider the distribution function as isotropic in the space of velocities, the integral-differential equation is transformed, as shown in [2], into a Fokker-Plank type differential equation having the form

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left\{ \frac{kT}{M} \frac{v^3}{\lambda(v)} \frac{\partial f}{\partial v} + \frac{mv^4}{M\lambda(v)} f \right\}, \qquad (1)$$

where λ is the length of electrons' free path. Let us assume, as in ref. [3],

that the effective crossection of electron-atom collisions is inversely proportional to the velocity of electrons, i.e.

$$\lambda = \lambda_0 \frac{v}{V} \left(V^2 = \frac{kT}{m}\right).$$

Such assumption is only valid for the Maxwellian interaction law, inversely proportional to the fifth power of distance.

At the initial moment the distribution function has the form

$$f(v, 0) = N\left(\frac{m}{2\pi kT_0}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT_0}\right]. \qquad (2)$$

We shall seek the solution of equation (1) in the form

$$f(v, t) = N\left(\frac{m}{2\pi k\vartheta(t)}\right)^{3/2} \exp\left[-\frac{mv^2}{2k\vartheta(t)}\right], \qquad (3)$$

where $\vartheta(t)$ is an unknown function subject to the initial condition

$$\vartheta(0) = T_0. \qquad (4)$$

Substituting (3) into (1), and effecting certain abbreviations, we shall obtain for $\vartheta(t)$ the usual relaxation equation

$$\frac{d\vartheta}{dt} = -\frac{1}{\alpha}(\vartheta - T), \qquad (5)$$

whose solution has the form

$$\vartheta - T = e^{-\frac{t}{\alpha}}(T_0 - T), \qquad (6)$$

where $\alpha = \frac{\lambda_0 M}{2mV}$ is the relaxation time. For an electron gas with $\lambda_0 = 6.5 \cdot 10^{-3}$ cm placed in a thermostat with temperature $\frac{3kT}{2} = 0{,}03$ *ev*

and $M = 30\,(1832)\,m$, $\alpha = 4.06 \cdot 10^{-5}$ sec [3].

As is well known, the Maxwellian potential has a limited application range. A question arises in connection with this, under what conditions, superimposed

on the interaction potential, the form of the Maxwellian distribution function will be preserved, if only in the first approximation? In order to obtain the corresponding criterion, let us make use of the perturbation theory. If at solution of equation (1) with an arbitrary function $\lambda(v)$, we take for the unperturbed solution the Maxwellian function (3), and if we consider as perturbation the right-hand part of (1), in which in place of $\frac{v}{\lambda(v)}$ there stands $\left(\frac{v}{\lambda(v)} - \frac{V}{\lambda_0}\right)$, the condition of applicability of the perturbation theory may be approximately written in the form

$$\left| \frac{T_0 - T}{T_0} \; \frac{z - z_M}{z_M} \right| \ll 1, \qquad (7)$$

where $z = \left(\frac{v}{\lambda}\right)_{cp}$, *), and where $z_M = \frac{V}{\lambda_0} \cdot \left(\frac{v}{\lambda}\right)_{cp}$ corresponds to a certain temperature from the (T_0, T) region, at which the second fraction in formula (7) is maximum. This condition gives precisely the indispensable criterion ensuring the preservation of the Maxwellian form of the distribution function at an arbitrary interaction potential.

It is seen from the condition (7) that for small deflections from the equilibrium position, the form of the Maxwellian function will be preserved for any reasonable potential of intermolecular reciprocal action, inasmuch as

$$\max \left| \frac{z - z_M}{z_M} \right| \sim 1.$$

As the temperature deflection increases, the requirements imposed on the potential become more rigid.

The obtained results are of interest to questions of kinetics, for example when examining the energy relaxation of progressive degrees of freedom in the mixture of light and heavy gases heated by a shock wave.

*) "cp" in the formula stands for "mean" or "average"

In conclusion, the author expresses his deep gratitude to Professor E. V. Stupochenko for the discussion of the results obtained.

***** END *****

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